ON POWER FUNCTION OF A SOMETIMES POOL TEST PROCEDURE IN A MIXED MODEL-I : A THEORETICAL INVESTIGATION

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SUMMARY

The present paper deals with a hypothesis testing problem based on conditional specification in a mixed model. A sometimes pool test procedure using two preliminary tests has been proposed for testing the hypothesis and the size and power of the test have been derived.

INTRODUCTION

1.1. Related Papers and Objective of the Present Study

Many investigations have been made in fixed and random models to study the power of test procedures incorporating one or two preliminary tests [Paull 8], Bechhofer [3], Bozivich, Bancroft and Hartley [4], Bancroft [2], Srivastava and Bozivich [10], Gupta and Srivastava [5], Saxena and Srivastava [9] and Mead, Bancroft and Han [6] but only a few studies have been made for a mixed model by Tailor and Saxena [12], [13], Bozivich, Bancroft and Hartley [4] and Agarwal and Gupta [1].

The present study has been made with a hypothesis testing problem based on conditonal specification in a mixed model situation arising from a split-plot in time experiment. Uncertainties involved in the model specification have been resolved by performing two preliminary tests and based on the outcome of these tests, a sometimes pool test has been proposed to test the hypothesis of no treatment differences. Expressions for power components of the test have been derived.

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1.2. The Model under Investigation and Conditional Specification

The mixed model understudy relates to an agricultural experiment called 'split-plot in time experiment' by Steel and Torrie [11]. Let us consider an experiment where yields are obtained on each plot for 't' cuttings of 's' alfalfa varieties in a randomised complete block design of 'r' blocks. Let Y_{ijk} denote the observation in the i^{th} block on the j^{th} variety where the k^{th} cutting was made. The sample observations may, therefore, be represented by the model

$$\gamma_{ijk}=\mu+\alpha_i+\beta_j+\delta_{ij}+\gamma_k+(\alpha\gamma)_{ik}+(\beta\gamma)_{jk}+\epsilon_{ijk}, \qquad \ldots(1.2.1)$$

$$(i=1, 2,\ldots\ldots, r, j=1, 2,\ldots\ldots, s, k=1, 2,\ldots\ldots, t)$$

where μ is the overall mean, β_j and γ_k are the variety and cutting effects, α_i the block effects and $(\alpha\gamma)_{ik}$ and $(\beta\gamma)_{jk}$ the interaction effects. The error terms δ_{ij} are assumed to be normally and independently distributed with mean 0 and variance σ_s^2 the common variance of the whole unit random component δ's; ϵ_{ijk} are normally and independently distributed with mean 0 and variance σ_ϵ^2, the common variance of the sub-unit random component ϵ's. Let us assume that the α's are random and the β's and γ's are fixed. Then we further assume that

$$\alpha_i \text{ are } NID\ (0, \sigma_\alpha^2),$$

$$\sum_j \beta_j=0,\ \sum_k \gamma_k=0,\ \sum_k (\beta\gamma)_{jk}=\sum_j (\beta\gamma)_{jk}=0,\ (\alpha\gamma)_{ik}$$

$$\text{are } NID\ (0,\ \sigma_{\alpha\gamma}^2),\ \sum_k (\alpha\gamma)_{ik}=0,\ \sum_i (\alpha\gamma)_{ik}\neq 0.$$

The analysis of variance corresponding to model (1.2.1) may be set up as in Table 1.1.

where σ_β^2, σ_γ^2 and $\sigma_{\beta\gamma}^2$ enclosed within parentheses refer to finite population variances and equal $\sum_j \beta_j^2/(s-1)$, $\sum_k \gamma_k^2/(t-1)$ and $\sum_j \sum_k (\beta\gamma)_{jk}^2/(s-1)(t-1)$ respectively.

Our main interest is to test the hypothesis concerning γ_k when nothing is known about the effects it produces in interaction with α_i or β_j. Now with $\sigma_\alpha^2 \geqslant o$, $\sigma_\beta^2 \geqslant o$, $\sigma_\delta^2 > o$, $\sigma_\epsilon^2 > o$, if uncertainties about

TABLE 1.1

Mixed Model ANOVA for a Split-plot in Time Experiment.

Source of Variation	*Degrees of Freedom*	*Expected Mean Square*
Blocks	r-1	$\sigma^2_\epsilon+st\sigma^2_\alpha$
Varieties	s-1	$\sigma^2_\epsilon+t\sigma^2_\delta+rt[\sigma^2_\beta]$
Error (a)	(r-1) (s-1)	$\sigma^2_\epsilon+t\sigma^2_\delta$
Cuttings	t-1	$\sigma^2_\epsilon+s\sigma^2_{\alpha\gamma}+rs[\sigma^2_\gamma]$
Cuttings × Blocks	(r-1) (t-1)	$\sigma^2_\epsilon+s\sigma^2_{\alpha\gamma}$
Cuttings × Varieties	(s-1) (t-1)	$\sigma^2_\epsilon+r[\sigma^2_{\beta\gamma}]$
Error (b)	(r-1) (s-1) (t-1)	σ^2_ϵ

interaction effects exists, *i.e.*, if $\sigma^2_{\alpha\gamma}\geqslant o$, $\sigma^2_{\beta\gamma}\geqslant o$, then (1.2.1) becomes

$$Y_{ijk}=\mu+\alpha_i+\beta_j+\delta_{ij}+\gamma_k+(\alpha\gamma)_{ik}+(\beta\gamma)_{jk}+\epsilon_{ijk},$$

for $$\sigma^2_{\alpha\gamma}>o,\ \sigma^2_{\beta\gamma}>o\ ; \qquad ...(1.2.2)$$

or $$Y_{ijk}=\mu+\alpha_i+\beta_j+\delta_{ij}+\gamma_k+(\alpha\gamma)_{ik}+\epsilon_{ijk},$$

for $$\sigma^2_{\alpha\gamma}>o,\ \alpha^2_{\beta\gamma}=o\ ; \qquad ...(1.2.3)$$

or $$Y_{ijk}=\mu+\alpha_i+\beta_j+\delta_{ij}+\gamma_k+(\beta\gamma)_{jk}+\epsilon_{ijk},$$

for $$\sigma^2_{\alpha\gamma}=o,\ \sigma^2_{\beta\gamma}>o\ ; \qquad ...(1.2.4)$$

or $$Y_{ijk}=\mu+\alpha_i+\beta_j+\delta_{ij}+\gamma_k+\epsilon_{ijk},$$

for $$\sigma^2_{\alpha\gamma}=o,\ \sigma^2_{\beta\gamma}=o. \qquad ...(1.2.5)$$

In this case, (1.2.1) is said to be an incompletely specified model or a model with conditional specification. If however. it is known with certainty that $\sigma^2_{\alpha\gamma}>o$, $\sigma^2_{\beta\gamma}>o$, then the appropriate model is (1.2.2) only and (1.2.1) is completely specified. Similarly, if it is known with certainty that $\sigma^2_{\alpha\gamma}=o$, $\sigma^2_{\beta\gamma}=o$, then the appropriate model is (1.2.5) only and again (1.2.1) is completely specified. The latter two situations are those of unconditional specification.

1.3. Description of the Problems and the Pooling Procedure

Let V_1, V_2, V_3, V_4 denote the mean squares respectively for the four components of variation viz., error (*b*), cuttings × varieties, cuttings × blocks, cuttings (Table 1.1) with corresponding degrees of freedon n_1, n_2, n_3, n_4 and expectations σ_1^2, σ_2^2, σ_3^2, σ_4^2. In the terminology of the area of conditional specification of the model V_1 and V_2 are called doubtful error mean squares, V_3 the error mean square and V_4 the treatment mean square. Accordingly, we may consider an abridged 'anova' table as shown below :

TABLE 1.2

Mixed Model Abridged ANOVA

Source of Variation	*Degrees of Freedom*	*Mean Square*	*Expected Mean Square*
Treatments	n_4	V_4	$\sigma_4^2=\sigma_3^2\left(1+\frac{2\lambda_4}{n_4}\right)$
True Error	n_3	V_3	$\sigma_3^2=\sigma_3^2$
Doubtful Error II	n_2	V_2	$\sigma_2^2=\sigma_1^2\left(1+\frac{2\lambda_2}{n_2}\right)$
Doubtful Error I	n_1	V_1	$\sigma_1^2=\sigma_1^2$

The doubtful error I and the true error mean squares are distributed as $\chi_i^2\,\sigma_i^2/n_i,(i=1,\ 3)$, where χ_i^2 is the central chi-square with n_i degrees of freedom and the remaining doubtful error II and the treatment mean squares are distributed as $\chi_2'^2\,\sigma_1^2/n_2$ and $\chi_4'^2\,\sigma_3^2/n_4$, where $\chi_2'^2$ and $\chi_4'^2$ are the non-central chi-squares with n_2 and n_4 degrees of freedom and the non-centrality parameters $\lambda_2=n_2\,(\sigma_2^2-\sigma_1^2)/2\sigma_1^2$ and $\lambda_4=n_4\,(\sigma_4^2-\sigma_3^2)/2\sigma_3^2$ respectively.

Now given $E(V_i)=\sigma_i^2\,(i=1,2,3,4)$ and σ_3^2 and/or $\sigma_2^2\geqslant\sigma_1^2$ we are interested in testing the hypothesis $H_0: \sigma_4^2=\sigma_3^2$ (*i.e.* $\lambda_4=0$) against the alternative $H_1: \sigma_4^2>\sigma_3^2$ (*i.e.* $\lambda_4>0$). Then if model (1.2.1) is assumed, it is clear that the appropriate test procedure for testing H_0 is to calculate F-statistic (*i.e,* $F=V_4/V_3$) and to reject it whenever a significant value of F is observed. Similarly, if model (1.2.5.) is assumed, our test criterion would be to calculate

$F=(n_1+n_2+n_3)V_4/(n_1V_1+n_2V_2+n_3V_3)$ and to reject H_0 if the calculated value of F turns out to be significant. In fact with these situations of unconditional specification of the model, the tests are uniquely determined. However, uncertainty might exist about the interactions and σ_3^2 and/or σ_2^2 might equal σ_1^2. In such a situation the uncertainty is first resolved by testing in succession the preliminary hypotheses $H_{01} : \sigma_3^2=\sigma_1^2$ and $H_{02} : \sigma_2^2=\sigma_1^2$ (*i.e.*, $\lambda_2=0$) against their corresponding alternatives $H_{11} : \sigma_3^2>\sigma_1^2$ and $H_{12} : \sigma_2^2>\sigma_1^2$ (*i.e.* $\lambda_2>0$). Depending on the outcome of these tests, appropriate tests are then devised to make a decision about H_0. Testing of preliminary hypotheses in succession and the final hypothesis thereafter leads to a sometimes pool test procedure.

In making the tests for H_{01} and H_{02}, it is likely that evidences may go against them and the hypotheses may be rejected in favour of pronounced interactions. Having noted that some of these interactions are present, an experimenter may proceed to examine the nature of these interactions. This may cause him to loose interest in the overall differences between cuttings, because the presence of cutting x blocks interaction implies that the effects of cuttings vary from one block to another and likewise, the presence of cuttings x varieties interaction entails the cutting differences not to be the same from variety to variety and so a detailed summary of the results is needed. However, the present study applies to cases where overall cutting differences are of interest regardless of the presence or absence of interactions.

The sometimes pool test procedure which we have proposed for testing H_0 consists in rejecting it if any one of the three mutually exclusive events occurs :

$$\left.\begin{array}{ll}(i) & V_3/V_1\geqslant F(n_3,n_1;\alpha_1),\ V_4/V_3\geqslant F(n_4,n_3;\alpha_2);\\ (ii) & V_3/V_1<F(n_3,n_1;\alpha_1),\ V_2/V_{13}\geqslant F(n_2,n_1+n_3;\alpha_3),\ V_4/V_{13}\geqslant F(n_4,n_1+n_3;\alpha_4);\\ (iii) & V_3/V_1<F(n_3,n_1;\alpha_1),\ V_2/V_{13}<F(n_2,n_1+n_3;\alpha_3),\ V_4/V_{123}\geqslant F(n_4,n_1+n_3;\alpha_5).\end{array}\right\}\quad ...(1.3.1)$$

where

$$V_{13}=\frac{n_1V_1+n_3V_3}{n_1+n_3},\quad V_{123}=\frac{n_1V_1+n_2V_2+n_3V_3}{n_1+n_2+n_3}$$

and $F(n_i,n_j;\alpha_k)$ refers to the upper $100\alpha_k\%$ point of the F-distribution with (n_i,n_j) degrees of freedom.

It may be remarked here that the split-plot in time experiment is not the only analysis of variance situation giving rise to sometimes pool test procedure (1.3.1). An analogous test situation may arise from a mixed model experiment based on a three-way classification with single observation per cell where one of the factors is random and the other two are fixed.

2. Derivation of the Power Function

2.1. Integral Expressions for Power components

Let P_1, P_2 and P_3 denote respectively the probabilities associated with the three mutually exclusive events (*i*), (*ii*) and (*iii*) of (1.3.1). Then the probability P of rejecting H_0 which is, in general, the power of the test will be the sum of the probabilities P_1, P_2 and P_3, *i.e.*,

$$P=\sum_{i=1}^{3} P_i, \qquad ...(2.1.1)$$

where

$$P_1=\text{Prob.}\ \{V_3/V_1 \geqslant F(n_3,n_1;\alpha_1),\ V_4/V_3 \geqslant (n_4,n_3;\alpha_2)\}; \qquad ...(2.1.2)$$

$$P_2=\text{Prob.}\ \{V_3/V_1 < F(n_3,n_1;\alpha_1),\ V_2/V_{13} \geqslant F(n_2,n_1+n_3;\alpha_3),\ V_4/V_{13} \geqslant F(n_4,n_1+n_3;\alpha_4)\}; \qquad ...(2.1.3)$$

$$P_3=\text{Prob.}\ \{V_3/V_1 < F(n_3,n_1;\alpha_1),\ V_2/V_{13} < F(n_2,n_1+n_3;\alpha_3),\ V_4V_{123} \geqslant F(n_4,n_1+n_2+n_3;\alpha_5)\}. \qquad ...(2.1.4)$$

The probabilities (2.1.2), (2.1.3) and (2.1.4) may be called the components of power of the proposed sometimes pool test procedure.

Patnaik (1949) suggested an approximation to the non-central chi-square according to which χ'^2_2 and χ'^2_4 are approximately distributed as $c_2\chi^2_{\nu_2}$ and $c_4\chi^2_{\nu_4}$, where $\chi^2_{\nu_2}$ and $\chi^2_{\nu_4}$ are the central chi-squares with corresponding degrees of freedom.

$$\nu_2=n_2+\frac{4\lambda_2^2}{n_2+4\lambda_2},\quad \nu_4=n_4+\frac{4\lambda_4^2}{n_4+4\lambda_4}$$

and the constant scale factors

$$c_2=1+\frac{2\lambda_2}{n_2+2\lambda_2},\quad c_4=1+\frac{2\lambda_4}{n_4+2\lambda_4};$$

Using Patnaik's approximation, the joint density of V_1, V_2, V_3 and V_4 can be written as follows :

$$g(V_1,V_2,V_3,V_4)=A\, V_1^{\frac{1}{2}n_1-1}V_2^{\frac{1}{2}\nu_2-1}\,V_3^{\frac{1}{2}n_3-1}V_4^{\frac{1}{2}\nu_4-1}$$

$$\exp\left\{-\tfrac{1}{2}\left(\frac{n_1V_1}{\sigma_1^2}+\frac{n_2V_2}{\sigma_1^2c_2}+\frac{n_3V_3}{\sigma_3^2}+\frac{n_4V_4}{\sigma_3^2c_4}\right)\right\},$$

where A is a constant independent of the V's.

If we introduce the new variables

$$u_1=\frac{n_4V_4}{n_3V_3c_4},\quad u_2=\frac{n_2V_2}{n_1V_1c_2},$$

$$u_3=\frac{n_3V_3}{n_1V_1\theta_{31}},\quad u_4=\frac{n_1V_1}{2\sigma_1^2},$$

where $$\theta_{31}=\frac{\sigma_3^2}{\sigma_1^2}$$

and integrate out u_4 over the range 0 to ∞, we get the joint density of u_1, u_2 and u_3 as

$$f(u_1,u_2,u_3)=A_1\frac{u_1^{\frac{1}{2}\nu_4-1}\,u_2^{\frac{1}{2}\nu_2-1}\,u_3^{\frac{1}{2}n_3+\frac{1}{2}\nu_4-1}}{(1+u_2+u_3+u_1u_3)^{\frac{1}{2}n_1+\frac{1}{2}\nu_2+\frac{1}{2}n_3+\frac{1}{2}\nu_4}},\qquad \ldots(2.1.5)$$

where

$$A_1=\frac{\Gamma(\frac{1}{2}n_1+\frac{1}{2}\nu_2+\frac{1}{2}n_3+\frac{1}{2}\nu_4)}{\Gamma(\frac{1}{2}n_1)\Gamma(\frac{1}{2}\nu_2)\Gamma(\frac{1}{2}n_3)\Gamma(\frac{1}{2}\nu_4)}.$$

The limits of integration of the new variables u_1, u_2 and u_3 corresponding to different components are as follows :

For P_1 : $a\leqslant u_3<\infty,\ 0\leqslant u_2<\infty,\ b\leqslant u_1<\infty$;

For P_2 : $0\leqslant u_3<a,\ c+fu_3\leqslant u_2<\infty,\ \dfrac{d+gu_3}{u_3}\leqslant u_1<\infty$;

For P_3 : $0\leqslant u_3<a,\ 0\leqslant u_2<c+fu_3,\ \dfrac{e+hu_2+mu_3}{u_3}\leqslant u_1<\infty$;

where

$$\left.\begin{aligned}
&a=u_1^0/\theta_{31},\ b=u_2^0/c_4,\ c=u_3^0/c_2,\ d=u_4^0/c_4\theta_{31}'\\
&e=u_5^0/c_4\theta_{31},\ f=\theta_{31}u_3^0/c_2,\ g=u_4^0/c_4,\ h=c_2u_5^0/c_4\theta_{31}'\\
&m=u_5^0/c_4;\\
&\text{and}\\
&u_1^0=\frac{n_3}{n_1}F(n_3,n_1;\alpha_1),\ u_2^0=\frac{n_4}{n_3}F(n_4,n_3;\alpha_2),\\
&u_3^0=\frac{n_2}{n_1+n_3}F(n_2,n_1+n_3;\alpha_3),\ u_4^0=\frac{n_4}{n_1+n_3}F(n_4,n_1\\
&\qquad+n_3;\alpha_4),\\
&u_5^0=\frac{n_4}{n_1+n_2+n_3}F(n_4,n_1+n_2+n_3;\alpha_5).
\end{aligned}\right\}\quad \ldots(2.1.6)$$

The integral expressions for the three components may, therefore, be written as follows :

$$P_1=\int_{u_3=a}^{\infty}\int_{u_2=0}^{\infty}\int_{u_1=b}^{\infty} f(u_1,u_2,u_3)\,du_3\,du_2\,du_1\ ; \qquad \ldots(2.1.7)$$

$$P_2=\int_{u_3=0}^{a}\int_{u_2=c+fu_3}^{\infty}\int_{u_1=\frac{d+gu_3}{u_3}}^{\infty} f(u_1,u_2,u_3)\,du_3\,du_2\,du_1; \qquad \ldots(2.1.8)$$

$$P_3=\int_{u_3=0}^{a}\int_{u_2=0}^{c+fu_3}\int_{u_1=\frac{e+hu_2+mu_3}{u_3}}^{\infty} f(u_1,u_2,u_3)\,du_3\,du_2\,du_1. \qquad \ldots(2.1.9)$$

2.2. Series Formulae for Power Components

The series formulae for different components of the sometimes pool test procedure have been obtained by evaluating the integral expressions (2.1.7), (2.1.8) and (2.1.9). These formulae are in terms of finite series because of even integer values assumed for n_1, ν_2, n_3 and ν_4.

Using (2.1.5) and (2.1.7) we may write P_1 as follows :

$$P_1=A_1\int_{u_3=a}^{\infty}\int_{u_2=0}^{\infty}\int_{u_1=b}^{\infty}\frac{u_1^{\frac{1}{2}\nu_4-1}\ u_2^{\frac{1}{2}\nu_2-1}\ u_3^{\frac{1}{2}n_3+\frac{1}{2}\nu_4-1}}{(1+u_2+u_3+u_1u_3)^{\frac{1}{2}n_1+\frac{1}{2}\nu_2+\frac{1}{2}n_3+\frac{1}{2}\nu_4}}\,du_3\,du_2\,du_1\ ; \qquad \ldots(2.2.1)$$

If in (2.2.1) we put $Z=(1+u_3+u_1u_3)/(1+u_2+u_3+u_1u_3)$ and integrate with respect to Z, we obtain

$$P_1 = A_2 \int_a^\infty \int_b^\infty \frac{u_1^{\frac{1}{2}\nu_1-1} \; u_3^{\frac{1}{2}n_3+\frac{1}{2}\nu_4-1}}{(1+u_3+u_1u_3)^{\frac{1}{2}n_1+\frac{1}{2}n_3+\frac{1}{2}\nu_4}} du_3\, du_1, \quad \ldots(2.2.2)$$

where

$$A_2 = \frac{\Gamma(\frac{1}{2}n_1+\frac{1}{2}n_3+\frac{1}{2}\nu_4)}{\Gamma(\frac{1}{2}n_1)\,\Gamma(\frac{1}{2}n_3)\,\Gamma(\frac{1}{2}\nu_4)}$$

Again, if we apply the transformation $y=(1+u_3)/(1+u_8+u_1u_3)$ in (2.2.2) and then integrate with respect to y, we get

$$P_1 = A_2 \sum_{I=0}^{\frac{1}{2}\nu_4-1} \frac{(-1)^I \binom{\frac{1}{2}\nu_4-1}{I}}{\frac{1}{2}n_1+\frac{1}{2}n_3+I} \int_a^\infty \frac{u_3^{\frac{1}{2}n_3-1}\,(1+u_3)^I}{\{1+(1+b)u_3\}^{\frac{1}{2}n_1+\frac{1}{2}n_3+I}} du_3 \quad \ldots(2.2.3)$$

The binomial expansion of $(1+u_3)^I$ and the transformation $x=1/\{1+(1+b)u_3\}$ applied in (2.2.3) give on simplification

$$P_1 = A_2 S_{IJ} \frac{Bx_1(\frac{1}{2}n_1+I-J,\, \frac{1}{2}n_3+J)}{(1+b)^{\frac{1}{2}n_3+J}}, \quad \ldots(2.2.4)$$

where

$$S_{IJ} = \sum_{I=0}^{\frac{1}{2}\nu_4-1} \frac{(-1)^I \binom{\frac{1}{2}\nu_4-1}{I}}{\frac{1}{2}n_1+\frac{1}{2}n_3+I} \sum_{J=0}^{I} \binom{I}{J},$$

$$Bx(p,q) = \int_0^x y^{p-1}\,(1-y)^{q-1}\, dy$$

and

$$x_1 = \frac{1}{1+(1+b)\,a}.$$

If we use the same method as in deriving P_1, we get the series formulae for P_2 and P_3 as follows :

$$P_2 = A_1 S_{IJKL}$$

$$S_M \frac{(1+d)^{K-M}\,(1+g)^M\, Bx_2(\frac{1}{2}n_3+L+M,\; \frac{1}{2}n_1+I-J+K-L-M)}{(1+c+d)^{\frac{1}{2}n_1+I-J+K-L-M}\;(1+f+g)^{\frac{1}{2}n_3+M+L}}, \quad \ldots(2.2.5)$$

$$P_3 = \frac{A_1 S_{IJKL}}{(1+h)^{\frac{1}{2}\nu_2+J}} \left\{ \frac{Bx_{31}(\frac{1}{2}n_3+L,\ \frac{1}{2}n_1+I-J-L)}{(1+e)^{\frac{1}{2}n_1+I-J-L}\ (1+m)^{\frac{1}{2}n_3+L}} \right.$$

$$\left. -S_M \frac{(1+e)^{K-M}(1+m)^M Bx_{32}(\frac{1}{2}n_3+L+M,\ \frac{1}{2}n_1+I-J+K-L-M)}{(1+c+e+ch)^{\frac{1}{2}n_1+I-J+K-L-M}\ (1+f+m+fh)^{\frac{1}{2}n_3+L+M}} \right\}, \quad \dots(2.2.6)$$

where

$$S_M = \sum_{M=0}^{k} \binom{k}{M},$$

$$S_{IJKL} = \sum_{I=0}^{\frac{1}{2}\nu_4-1} \frac{(-1)^I \binom{\frac{1}{2}\nu_4-1}{I}}{\frac{1}{2}n_1+\frac{1}{2}\nu_2+\frac{1}{2}n_3+I} \sum_{J=0}^{I} \binom{I}{J}$$

$$\sum_{K=0}^{\frac{1}{2}\nu_2+J-1} \frac{(-1)^K \binom{\frac{1}{2}\nu_2+J-1}{K}}{\frac{1}{2}n_1+\frac{1}{2}n_3+I-J+K} \sum_{L=0}^{I-J} \binom{I-J}{L},$$

$$x_2 = \frac{a(1+f+g)}{1+c+d+a(1+f+g)},$$

$$x_{31} = \frac{a(1+m)}{1+e+a(1+m)},$$

$$x_{32} = \frac{a(1+f+m+fh)}{1+c+e+ch+a(1+f+m+fh)}.$$

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